ITERATIVE SOLUTION OF DIFFUSION EQUATION WITH CRANK-NICOLSON SCHEME USING PAOR METHOD

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Abstract: In this paper, we solve one-dimensional diffusion equation with second order Crank-Nicholson scheme by PAOR method and this method compared with other methods discussed in this paper through some numerical examples.

Keywords: Finite Difference Method, Crank-Nicolson scheme, AOR, SOR, Gauss-Seidel, Jacobi.

Introduction

Let us consider one dimensional diffusion equation

$$\frac{\partial U}{\partial t} = D\frac{\partial^2 U}{\partial x^2} \quad 0<x<L,\ 0<t<T \qquad \ldots (1.1)$$

Partitioning the spacial interval $[0,L]$ and temporal interval $[0,T]$ into respective finite grids as

$$x_i = i\Delta x \quad i=0,1,2,3\ldots N \quad \text{where} \quad L/N=\Delta x \qquad \ldots(1.2)$$

$$t_j = j\Delta t \quad j=0,1,2,3\ldots M \quad \text{where} \quad T/M=\Delta t \qquad \ldots(1.3)$$

Denoting the numerical solution of $U(x,t)$ as $U_{i,j}=U(x_i,t_j)$

The Second order Crank-Nicolson scheme of equation (1.1) is

$$\frac{U_{i,j+1}-U_{i,j}}{\Delta t} = \frac{D}{2}\left[\frac{U_{i+1,j}-2U_{i,j}+U_{i-1,j}}{(\Delta x)^2}+\frac{U_{i+1,j+1}-2U_{i,j+1}+U_{i-1,j+1}}{(\Delta x)^2}\right] \qquad \ldots (1.4)$$

$$\Rightarrow 2U_{i,j+1}-2U_{i,j} = \frac{D\Delta t}{(\Delta x)^2}\left[U_{i+1,j}-2U_{i,j}+U_{i-1,j}+U_{i+1,j+1}-2U_{i,j+1}+U_{i-1,j+1}\right]$$

$$\Rightarrow -\lambda U_{i-1,j+1}+2(1+\lambda)U_{i,j+1}-\lambda U_{i+1,j+1} = \lambda U_{i-1,j}+2(1-\lambda)U_{i,j}+\lambda U_{i+1,j} \qquad \ldots (1.5)$$

where $\lambda = D\Delta t/(\Delta x)^2$

From (1.5), we get for first time level; $i=1,2,3\ldots N-1,\ j=0$

$$-\lambda U_{0,1}+2(1+\lambda)U_{1,1}-\lambda U_{2,1}=\lambda U_{0,0}+2(1-\lambda)U_{1,0}+\lambda U_{2,0}$$

$$-\lambda U_{1,1}+2(1+\lambda)U_{2,1}-\lambda U_{3,1}=\lambda U_{1,0}+2(1-\lambda)U_{2,0}+\lambda U_{3,0}$$

$$\vdots$$

$$-\lambda U_{N-2,1}+2(1+\lambda)U_{N-1,1}-\lambda U_{N,1}=\lambda U_{N-2,0}+2(1-\lambda)U_{N-1,0}+\lambda U_{N,0}$$

For second time level; $i=1,2,3\ldots N-1,\ j=1$

